Strange inequality.
https://www.linkedin.com/feed/update/urn:li:activity:6709501333495435264
Let x, y and z be positive real numbers, prove that
$(x+y-z)\left(\frac{3}{x+y}-\frac{1}{y+z}-\frac{1}{z+x}\right) \leq \frac{1}{2}$.
Solution by Arkady Alt, **San Jose**, **California**, **USA**.
Let $p := x+y$ and $q := xy \leq \frac{(x+y)^2}{4} = \frac{p^2}{4}$. Assume also $z = 1$ (due homogenety of the inequality). Then the inequality becomes $(p-1)\left(\frac{3}{p}-\frac{1}{y+1}-\frac{1}{1+x}\right) \leq \frac{1}{2} \Leftrightarrow$
(1) $(p-1)\left(\frac{3}{p}-\frac{2+p}{q+p+1}\right) \leq \frac{1}{2}$.
Since $\frac{2+p}{q+p+1} \geq \frac{2+p}{\frac{p^2}{4}+p+1} = \frac{4}{p+2}$ then $(p-1)\left(\frac{3}{p}-\frac{2+p}{q+p+1}\right) \leq$
$(p-1)\left(\frac{3}{p}-\frac{4}{p+2}\right) = \frac{(6-p)(p-1)}{p(p+2)}$ and also we have $\frac{(6-p)(p-1)}{p(p+2)} \leq \frac{1}{2} \Leftrightarrow$
$2(6-p)(p-1) \leq p(p+2) \Leftrightarrow 0 \leq p(p+2)-2(6-p)(p-1) \Leftrightarrow 0 \leq 3(p-2)^2$.